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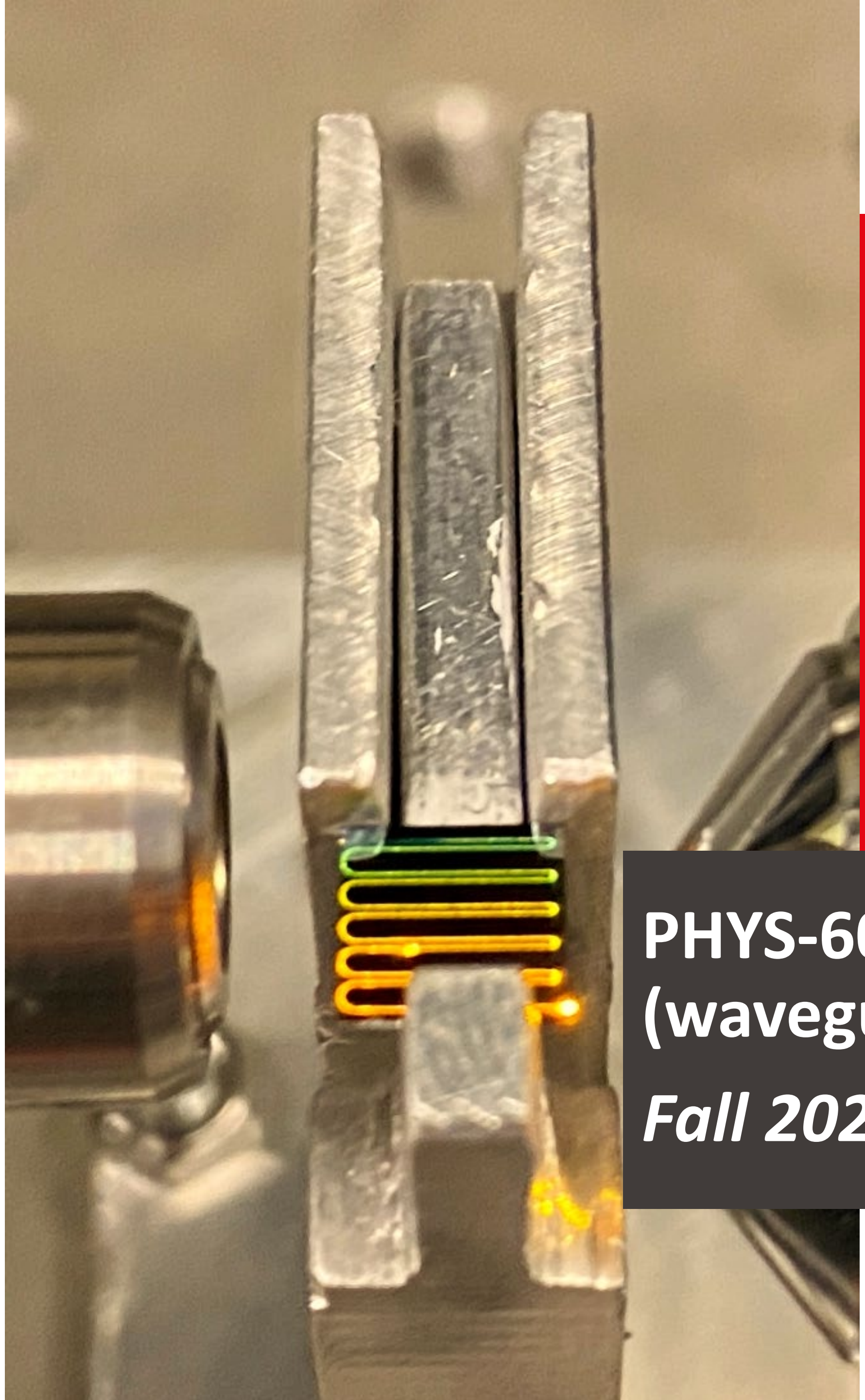

Optical Kerr effect

Module 4 – Solitons

PHYS-607 : Nonlinear fibre (waveguide) optics

Fall 2022

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Ecole Polytechnique Fédérale de Lausanne

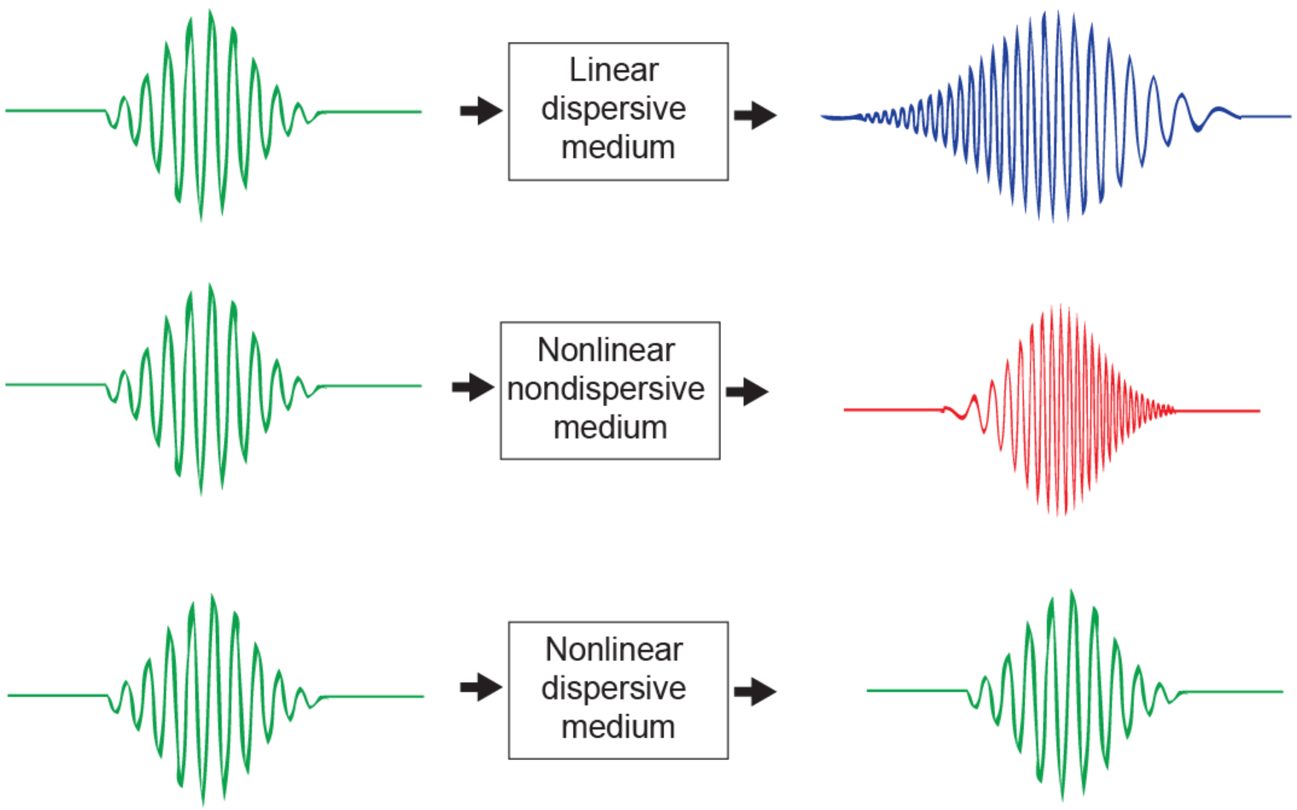
Soliton discovery

The Scottish engineer John Scott Russell (1808–1882) recalls the first observation of a soliton, while horseback riding near a canal:

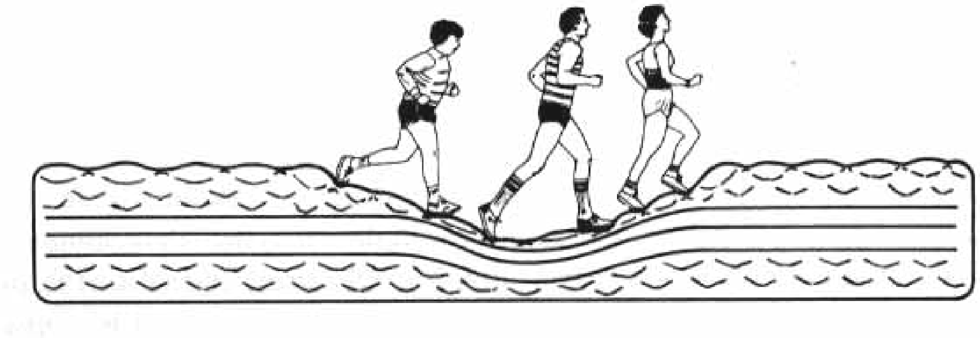
”…rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed…

… and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation”

Soliton ingredients



Analogy...



Runners create a moving valley that pulls slower runners and retards faster ones. (otherwise, the faster runners would move ahead and the slower behind)

It is possible to demonstrate that for such a medium a pulse exist which will propagate *without its envelope being modified*. Moreover it is a stable solution.

Soliton equation

Start with NLSE: $$\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} = j\gamma|A|^2 A \Longrightarrow j\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma|A|^2 A = 0$$

The equation takes normalized form using: $\tau = \frac{T}{T_0}, \xi = \frac{z}{L_D}, U = \frac{A}{\sqrt{P_0}}, N^2 = \gamma P_0 \frac{T_0^2}{|\beta_2|}$

$$j\frac{\partial U}{\partial \xi} - \frac{\text{sgn}(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} + N^2|U|^2 U = 0$$

Finally for anomalous dispersion ($\beta_2 < 0$), and normalizing $u \equiv NU$ we therefore get:

$$j\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0$$

This form of the NLS equation can be solved exactly !

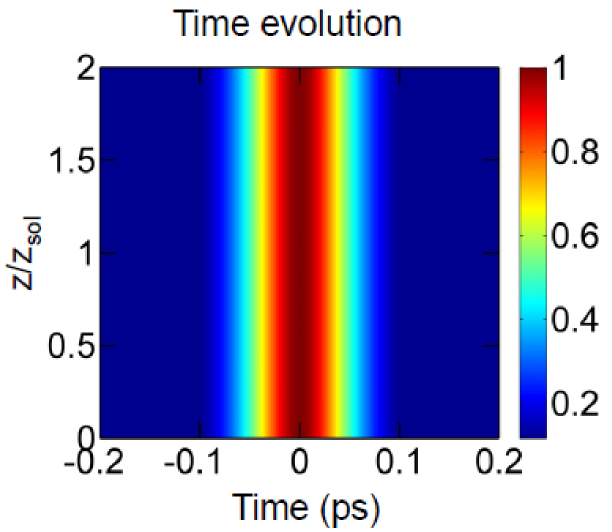
Soliton of order $N = 1$ – fundamental soliton

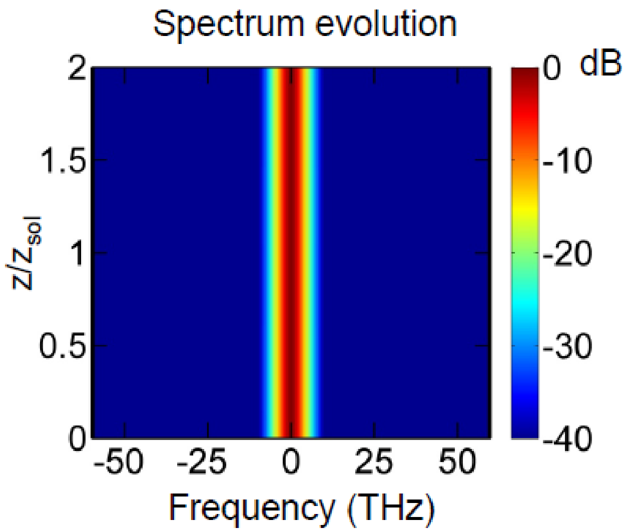
On can show that a solution of the previous equation is given by:

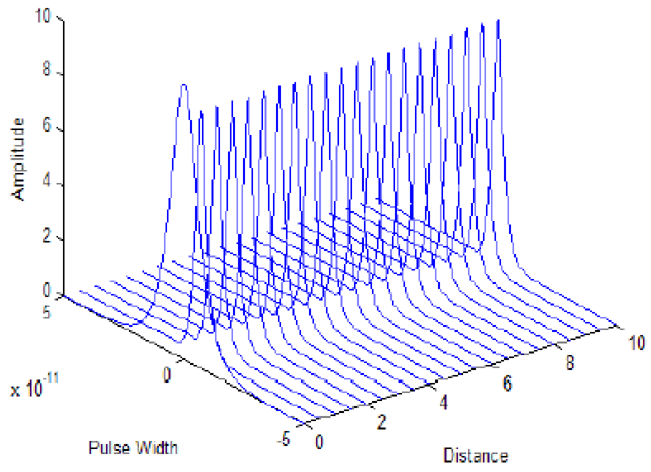
$$u(\xi,\tau) = \mathrm{sech}(\tau)\exp\left(j\frac{\xi}{2}\right)$$

$$A(z,T) = \sqrt{P_0}\,\mathrm{sech}\left(\frac{T}{T_0}\right)\exp\left(j\frac{z}{2L_D}\right)$$

- When such pulse is launched into a waveguide, it’s shape remains unchanged along propagation
- Above threshold, the pulse adjusts its shape & width as to attain sech^2 intensity profile for $z >> L_D$





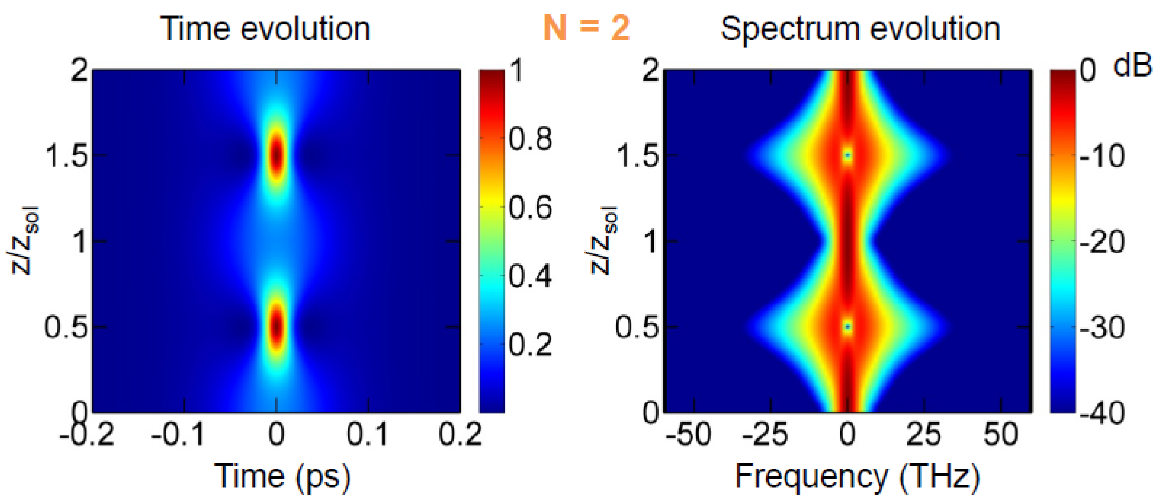


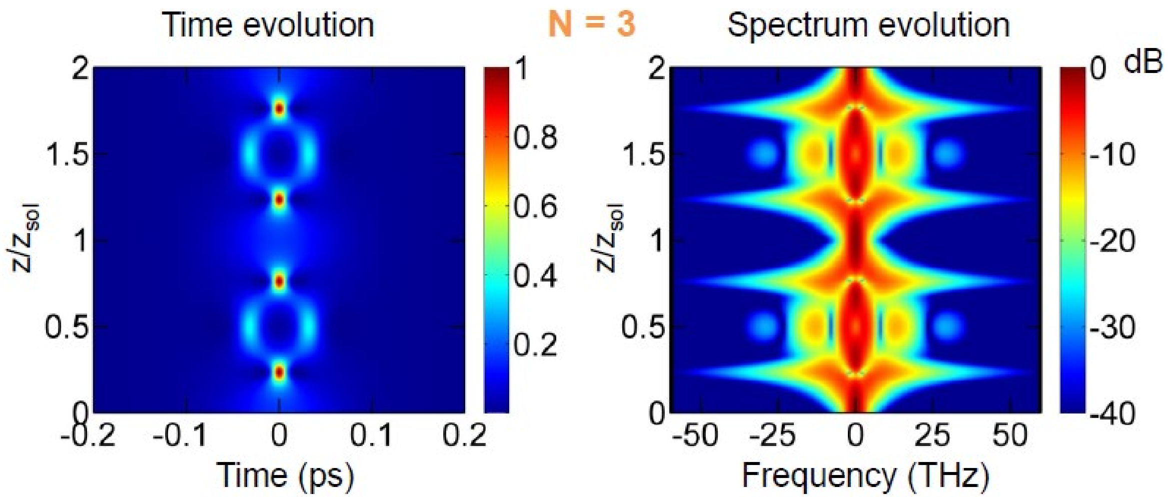
Soliton of order N > 1 – higher order solitons

Higher-order solitons are periodic upon propagation: $A(z + z_{sol}, T) = A(z, T)$

Requirements
$$\begin{cases} A(O,T) = \sqrt{P_0}\,\mathrm{sech}\left(\dfrac{T}{T_0}\right) \\ N = \sqrt{\gamma P_0 \dfrac{T_0^2}{|\beta_2|}} = 2,3,4\ \ldots \end{cases}$$

$$z_{sol} = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|}$$ *Soliton period*





Triggering a soliton

A soliton has a hyperbolic secant profile with given energy

- Can be generated in anomalous dispersion
- It is a stable solution and can form from an approximately correct pulse shape
- It is relatively stable and robust to small perturbations: excess power, losses …

The critical parameter is the soliton number N: $$N^2 = \frac{L_D}{L_{NL}} = \gamma P_0 \frac{T_0^2}{|\beta_2|}$$

If $N < 1$:

- Soliton cannot be supported, pulse propagates as normal and disperses

For $N > 1$:

- If the power decreases, a lower order soliton will form or the soliton width will increase
- If the power increases, a higher order soliton could form or the soliton width will decrease

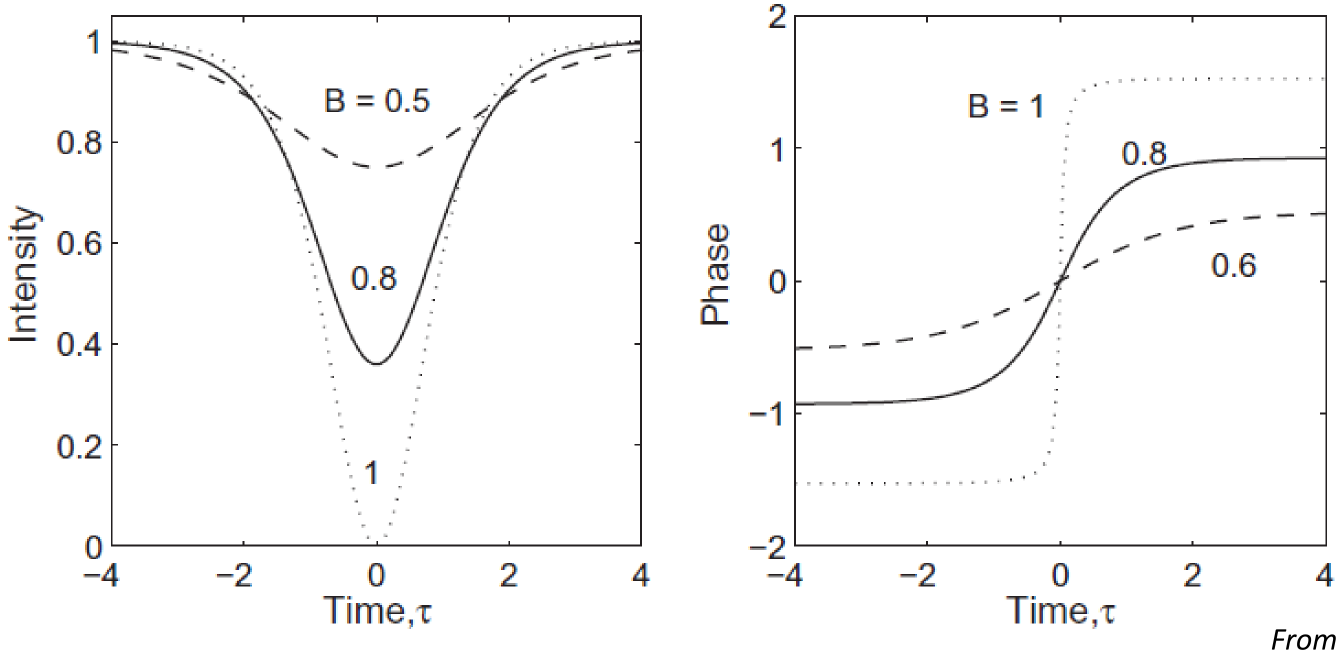
What about normal dispersion ?

Pulse like soliton form in anomalous dispersion –they are called *bright solitons*

- Bright solitons are obtained when $\gamma\beta_2 < 0$ (i.e. always in the anomalous dispersion region of a silica fiber since $\gamma > 0$).

It is also possible to get soliton like structures when propagating in the normal dispersion.

- They are called *dark solitons* and appear in the form of a dip within a CW background



From Agrawal

Quiz

A Gaussian pulse with a full width half maximum of T_{FWHM} = 10 ps is launched into an optical fiber with a dispersion parameter of D = 15.7 ps/nm-km a nonlinear coefficient γ = 2 $W^{-1}km^{-1}$.

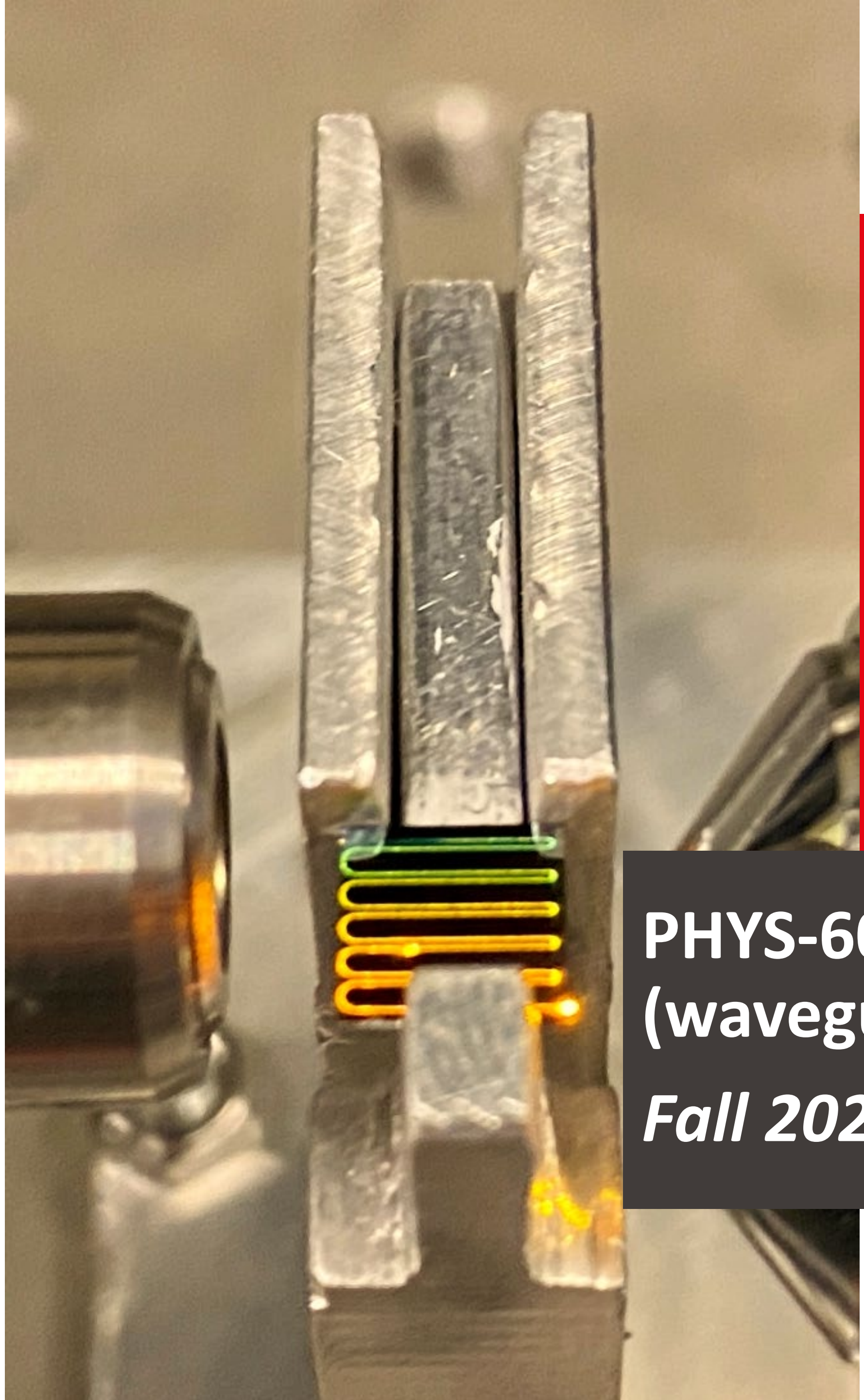
What should be the peak power of the pulse in order to excite a soliton of order 2 inside the fiber?

a) 1.1 W

b) 400 mW

c) 872 mW

d) 314 mW



Optical Kerr effect

Module 5 – Modulation instability

PHYS-607 : Nonlinear fibre (waveguide) optics

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Modulation instability

Many nonlinear systems exhibit an instability

- It is another result from the interplay between nonlinearity and dispersion.

What happens when a CW light wave propagates in a nonlinear and dispersion medium ?

- CW is never really CW
- In anomalous dispersion, a CW light (or quasi-CW) can breakup into a train of ultrashort pulses

This phenomenon is referred to as **modulation instability (MI)**

A few remarks about modulation instability


Contents lists available at ScienceDirect
Physica D
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ELSEVIER

Modulation instability: The beginning

V.E. Zakharov[a,b], L.A. Ostrovsky[c,d,*]

A possible definition:

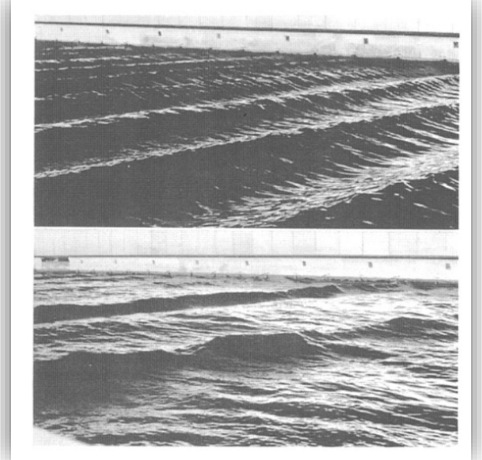
- '… amplification of a weak perturbation at the expense of a strong wave …'

Studied in various diverse fields

- Plasma physics, Bose-Einstein condensate, solid state physics…

Was also observed in different fields of physics

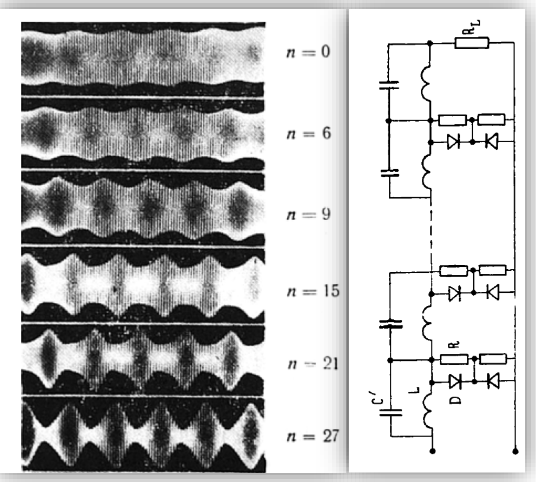
Hydrodynamics (1967)



Wavelength = 2 m

Benjamin, T. B. & Feir, J. E. The Disintegration of Wave Trains on Deep Water Part 1. Theory. *Journal of Fluid Mechanics* **27**, 417–430 (1967).

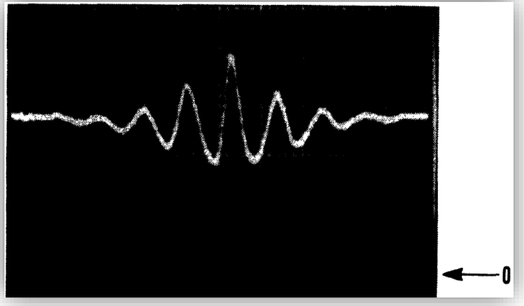
Dielectric media (1972)



Wavelength = 10^{-6} m

L.A. Ostrovskii, L.V. Soustov, Self-modulation of electromagnetic waves in nonlinear transmission lines, Izvestiya VUZ, Radiofizika 15 (1972) 242248.

Optics(1986)



Wavelength = 1300x10^{-9} m

Tai, K., Hasegawa, A. & Tomita, A. Observation of modulational instability in optical fibers. *Phys. Rev. Lett.* **56,** 135 (1986).

Linear stability analysis

Let's consider propagation inside a waveguide, neglecting loss and higher order dispersion:

$$j\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

Let's start by considering the input field A as a **perfect CW** (no dependence on T) and assuming it remains time independent during propagation. We get the following solution:

$$\bar{A} = \sqrt{P_0}\exp(j\phi_{NL}) \qquad \phi_{NL} = \gamma P_0 z$$

Is this steady state solution stable against small perturbations ? Let's perturb it:

$$A = \left(\sqrt{P_0} + a(z,T)\right)\exp(j\phi_{NL})$$

Linear stability analysis

We insert the perturbed solution $A = \left(\sqrt{P_0} + a(z,T)\right)\exp(j\phi_{NL})$ into the NLSE.

We linearize in a (neglecting terms in a^2 and a^3):

$$j\frac{\partial a}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2} + \gamma P_0(a + a^*) = 0$$

The solution has the form:

$$a(z,T) = a_1 \exp[j(\mathrm{K}z - \Omega T)] + a_2 \exp[-j(\mathrm{K}z - \Omega T)]$$

frequency of the perturbation

wavenumber of the perturbation

- If K is real $\Rightarrow$ phase
- If K is imaginary $\Rightarrow$ exponential solution type

Linear stability analysis

Set of 2 equations (real + imaginary part)

- a_1 and a_2 have a nontrivial solution only when K and Ω satisfy the following dispersion relation

$$\mathrm{K} = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + \mathrm{sgn}(\beta_2)\,\Omega_c^2} \quad \text{with } \Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|} = \frac{4}{|\beta_2|L_{NL}}$$

<u>For $\beta_2 > 0$ (normal dispersion):</u>

$$\mathrm{K} = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + \Omega_c^2}$$

K is real for any $\Omega \Rightarrow$ the **CW wave is stable against perturbation**

<u>For $\beta_2 < 0$ (anomalous dispersion):</u>

$$\mathrm{K} = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 - \Omega_c^2}$$

K is imaginary for $\Omega_c > |\Omega| \Rightarrow$ the **CW wave is unstable against perturbation**

Linear stability analysis

The gain in power experienced by the small perturbation reads:

$$g(\Omega) = 2\mathrm{Im}(\mathrm{K})$$

$$g(\Omega) = |\beta_2\Omega|\sqrt{\Omega_c^2 - \Omega^2}\ for\ \ |\Omega| < \Omega_C$$

The maximum gain is for a frequency given by:

$$\Omega_{max} = \pm\frac{\Omega_C}{\sqrt{2}} = \pm\sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

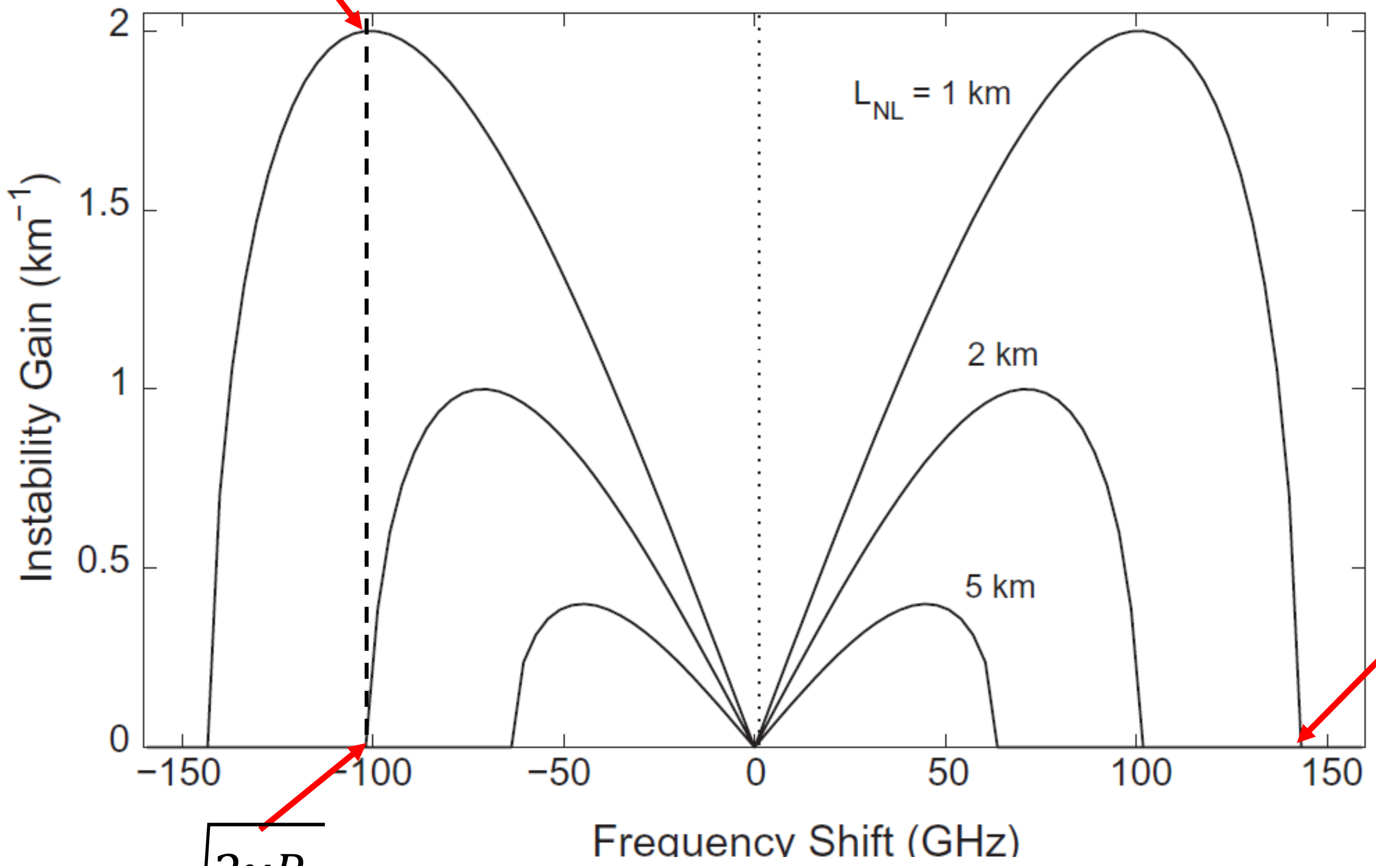
$$g_{max} = \frac{1}{2}|\beta_2|\Omega_C^2 = 2\gamma P_0$$

Try to go through this analysis

Graphical representation

$g_{max} = 2\gamma P_0$

β_2 = -5 ps²/km

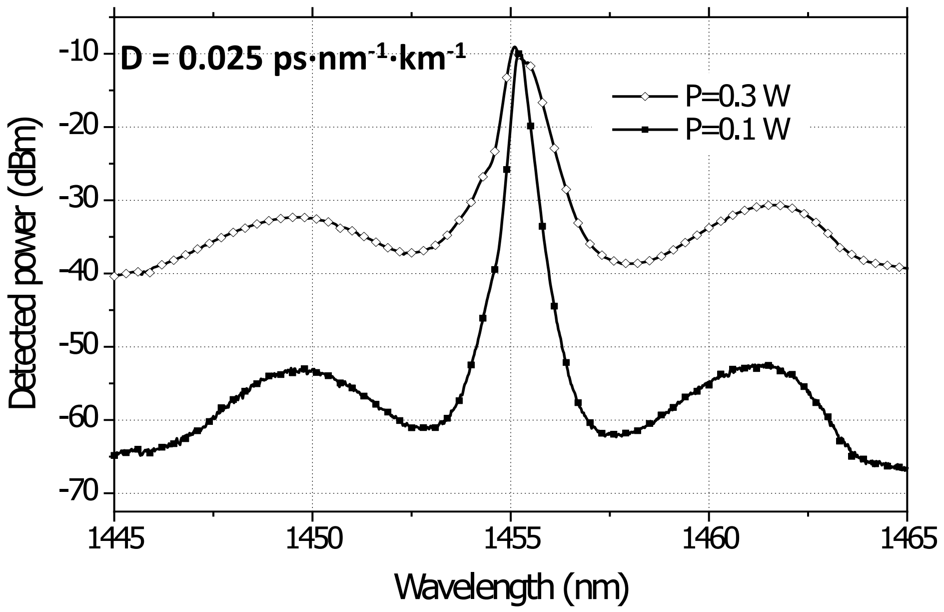


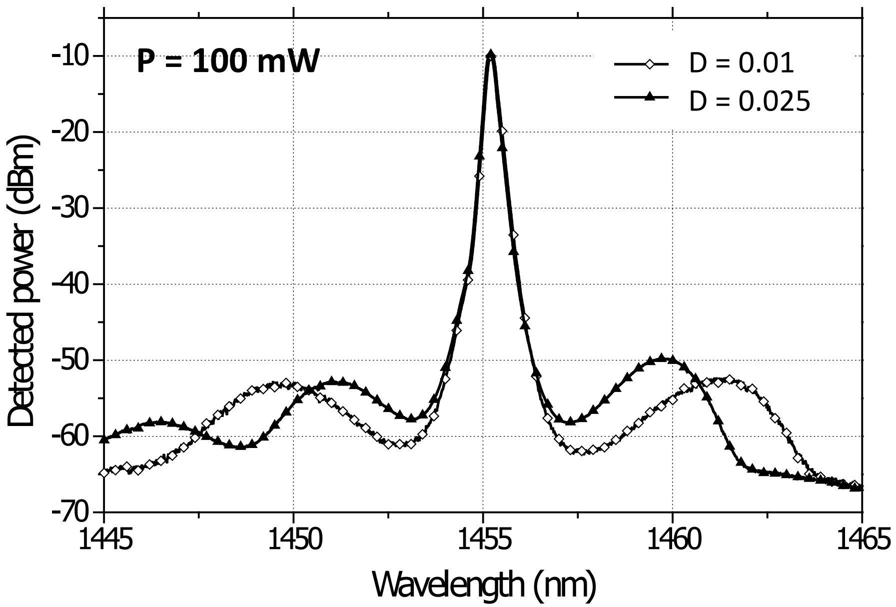
$$\Omega_c = \sqrt{\frac{4}{|\beta_2| L_{NL}}}$$

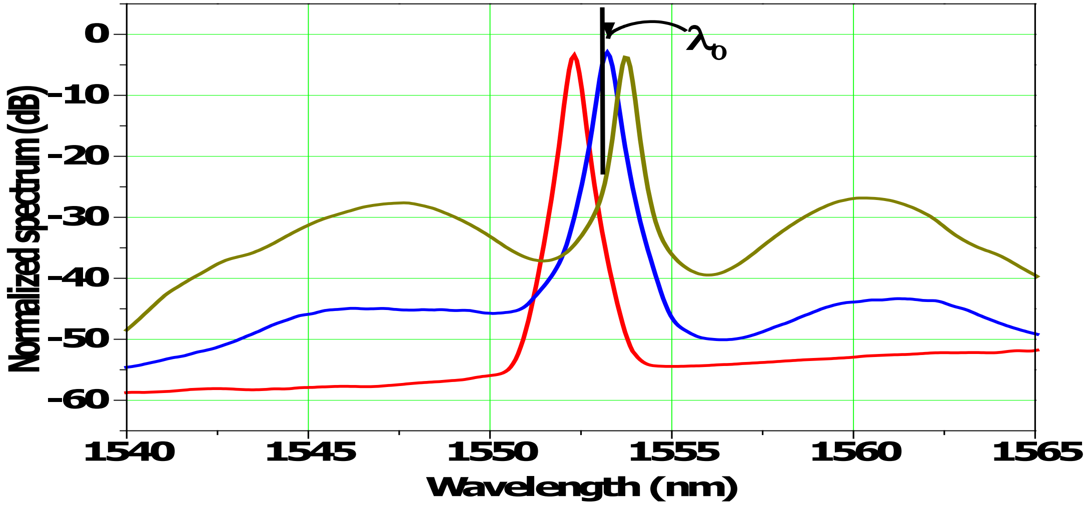
$$|\Omega_{max}| = \sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$

This is easily simulated: try with varying optical power P_0 or varying dispersion values D (β_2)

Experimental observation of modulation instability







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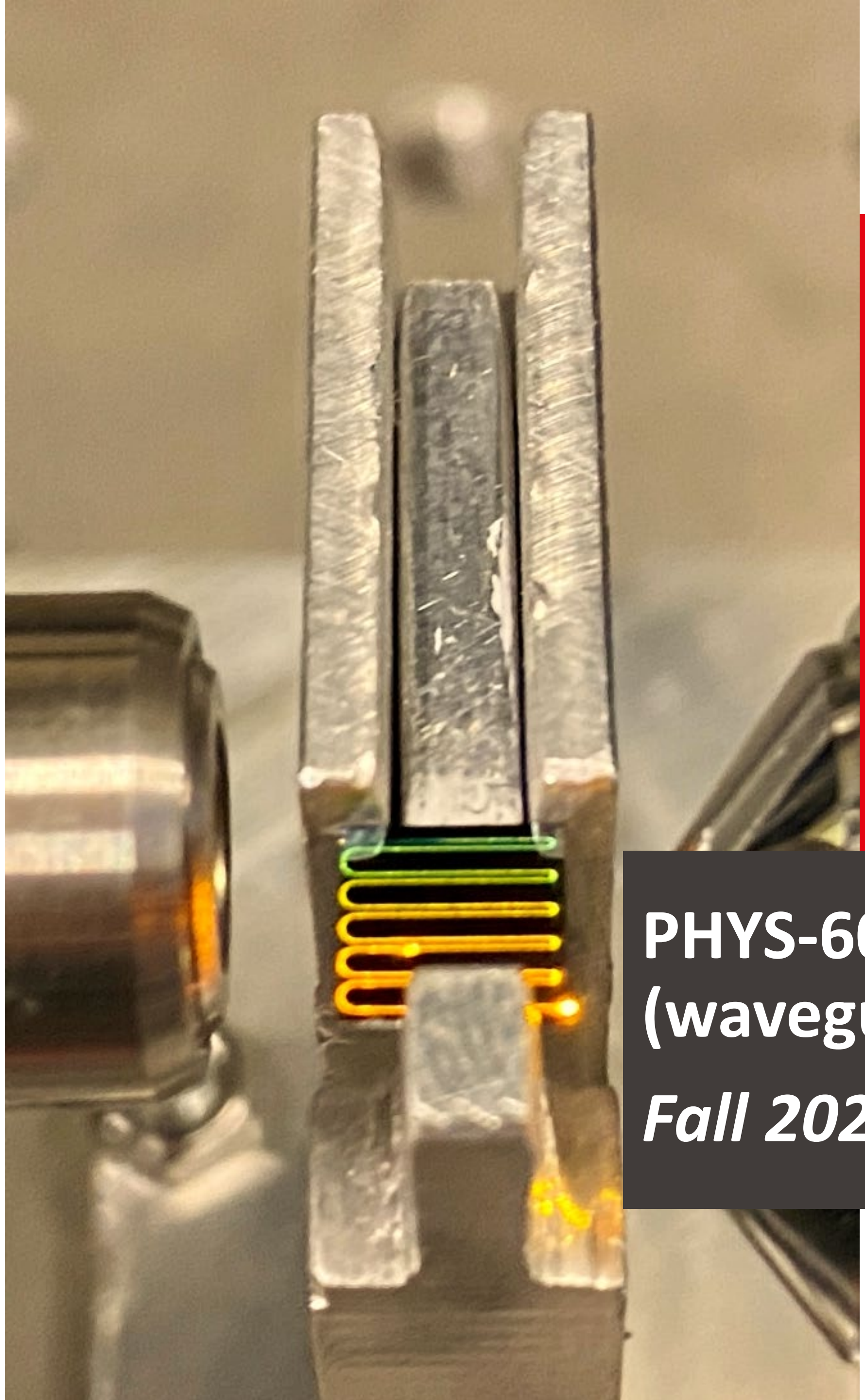

Optical Kerr effect

Module 6 – Solving the NLSE

PHYS-607 : Nonlinear fibre (waveguide) optics

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Recall nonlinear pulse propagation

Reminder

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = j\gamma|A|^2 A - \frac{\alpha}{2}A$$

Changing the static time frame of reference for one that moves at the group velocity ($v_g = 1/\beta_1(\omega)$)

$$\frac{\partial A}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} = j\gamma|A|^2 A - \frac{\alpha}{2}A$$

In silica this equation is valid when β_2 dominates over higher order dispersion terms and when $T_{\text{FWHM}} > 10$ ps

Generalized NLSE – valid down to few optical cycles

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha}{2}A}_{\text{Loss}} + \sum_{k\geq 2} \underbrace{\frac{j^{k+1}}{k!}\beta_k \frac{\partial^k A}{\partial T^k}}_{\text{Dispersion}} + j\left(\gamma + \underbrace{j\gamma_1 \frac{\partial}{\partial T}}_{\text{Self-steepening}}\right)\left[A(z,T) \underbrace{\int_0^\infty R(t')|A(z,T-t')|^2 dt'}_{\text{SPM, FWM, Raman}}\right]$$

Includes the frequency dependence of γ: $\gamma(\omega) \approx \gamma(\omega_0) + \gamma_1(\omega - \omega_0) + \cdots$

- γ_1 term forces v_g to be depend on intensity and leads to the phenomenon of *self steepening*

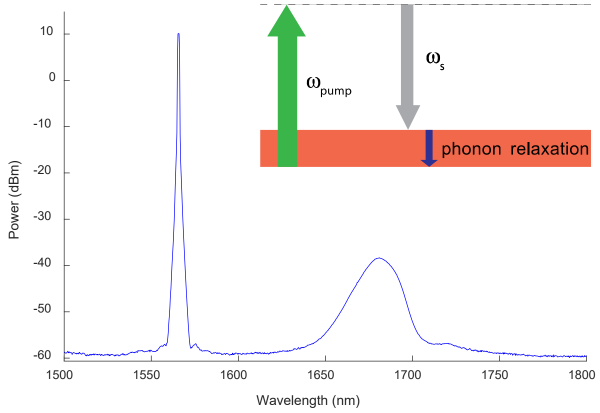
➡ *You can show that* $\frac{\gamma_1}{\gamma(\omega_0)} = \frac{1}{\omega_0} + \frac{1}{n_2}\left(\frac{dn_2}{d\omega}\right)\Big|_{\omega=\omega_0} - \frac{1}{A_{eff}}\left(\frac{dA_{eff}}{d\omega}\right)\Big|_{\omega=\omega_0}$

Generalized NLSE – valid down to few optical cycles

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha}{2}A}_{\text{Loss}} + \sum_{k\geq 2}\underbrace{\frac{j^{k+1}}{k!}\beta_k\frac{\partial^k A}{\partial T^k}}_{\text{Dispersion}} + j\left(\gamma + \underbrace{j\gamma_1\frac{\partial}{\partial T}}_{\text{Self-steepening}}\right)\left[\underbrace{A(z,T)\int_0^{\infty} R(t')|A(z,T-t')|^2 dt'}_{\text{SPM, FWM, Raman}}\right]$$

The integral accounts for the energy transfer resulting from intrapulse Raman scattering, related to the delayed nature of the Raman (vibrational response):

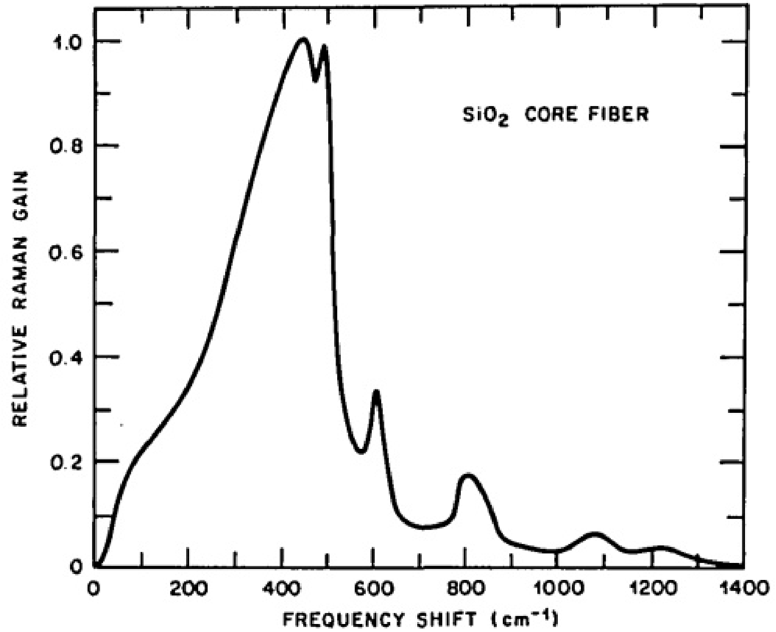
$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \qquad \int_0^{\infty} R(t)dt = 1$$



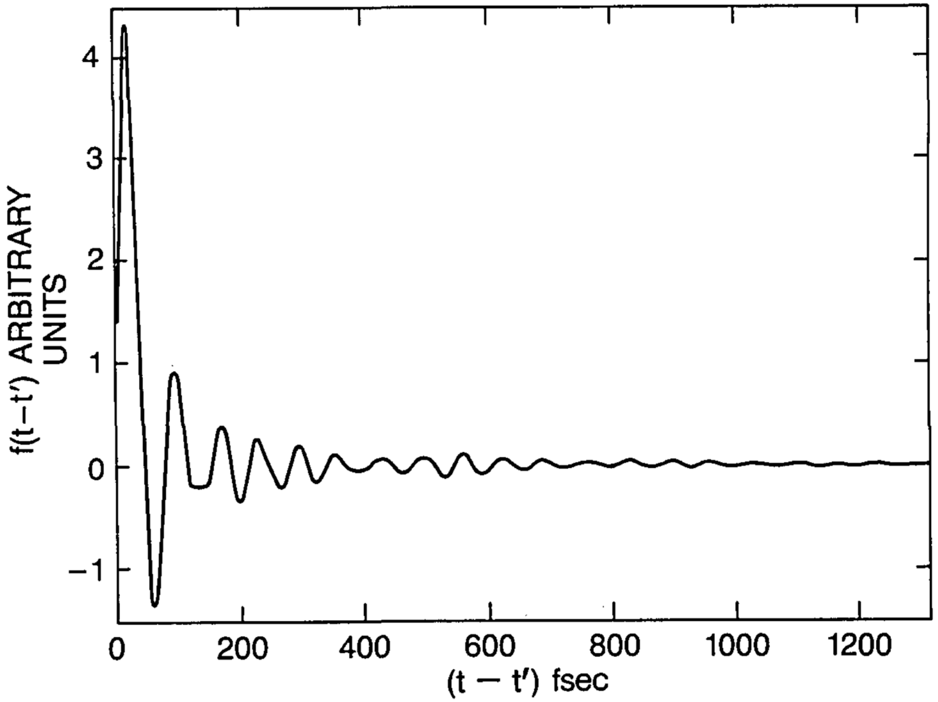
- f_R is the fractional contribution to the delayed Raman response to P_{NL}.
- $h_R(t)$ is the form of the Raman response function, set by the vibrations of silica molecules induced by an optical field

Raman response function

$h_R(t)$ is obtained by an indirect experimental approach as the Raman gain spectrum is easy to obtain experimentally



R.H. Stolen et al. Journal of the Optical Society of America B 1(4) (1984)



R.H. Stolen et al. Journal of the Optical Society of America B 6(6) (1989)

$$g_R(\Delta\omega) = \frac{\omega_0}{cn(\omega_0)} f_R \chi^{(3)} \mathrm{Im}\left[\tilde{h}_R(\Delta\omega)\right]$$

$\mathrm{Re}\left[\tilde{h}_R(\Delta\omega)\right]$ can be obtained through Kramers-Kronig relation

Generalized NLSE – further simplications for pulse with a few optical cycles

$$|A(z,T-t')|^2 \approx |A(z,T)|^2 - t'\frac{\partial |A(z,T)|^2}{\partial T}$$

$$\frac{\gamma_1}{\gamma(\omega_0)} =\approx \frac{1}{\omega_0}$$

$$T_R \equiv \int_0^\infty tR(t)dt$$

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + \sum_{k\geq 2}\frac{j^{k+1}}{k!}\beta_k\frac{\partial^k A}{\partial T^k} + j\gamma\left[A|A|^2 + \frac{j}{\omega_0}\frac{\partial}{\partial T}(A|A|^2) - T_R A\frac{\partial |A|^2}{\partial T}\right]$$

$$\frac{\partial A}{\partial z} = (\widehat{D}_T + \widehat{N}_T)A$$

$$\widehat{D}_T = -\frac{\alpha}{2} + \sum_{k\geq 2}\frac{j^{k+1}}{k!}\beta_k\frac{\partial^k}{\partial T^k}$$

Dispersion operator

$$\widehat{N}_T = j\gamma\left[|A|^2 + \frac{j}{\omega_0}\frac{1}{A}\frac{\partial}{\partial T}(A|A|^2) - T_R\frac{\partial |A|^2}{\partial T}\right]$$

Nonlinear operator

Numerical method – split-step Fourier method

In general dispersion and nonlinearity act together during propagation

- The split-step Fourier method gives an approximate solution
- Assumes that over a small propagation distance h, the two effects act *independently* and *successively*

$$A(z+h,T) = \exp\left[h\left(\hat{D}_T + \hat{N}_T\right)\right] A(z,T)$$

$$A(z+h,T) \cong \exp\left[h\hat{D}_T\right] \exp\left[h\hat{N}_T\right] A(z,T)$$

While $\hat{N}_T$ applies well in the time domain, $\hat{D}_T$ is most easily applied in the frequency domain:

$$\hat{D}_T = -\frac{\alpha}{2} + \sum_{k\geq 2} \frac{j^{k+1}}{k!} \beta_k \frac{\partial^k}{\partial T^k} \Longrightarrow \hat{D}_\omega = -\frac{\alpha}{2} + \sum_{k\geq 2} \frac{j^{k+1}}{k!} \beta_k \left(-j(\omega-\omega_0)\right)^k$$

- We have that : $\exp\left[h\hat{D}_T\right] A(z,T) = iFT\left\{\exp\left(h\hat{D}_\omega\right) FT[A(z,T)]\right\}$

Numerical method – split-step Fourier method

$$A(z+h,T) \cong iFT\Big\{\exp\big(h\widehat{D}_\omega\big)\, \underbrace{FT\Big[\underbrace{\exp\big(h\widehat{N}_T\big)\, A(z,T)}_{\text{Nonlinear operator in time domain}}\Big]}_{\text{Dispersion operator in frequency domain}}\Big\}$$

OR

$$\tilde{A}(z+h,\omega) \cong FT\Big\{\exp\big(h\widehat{N}_T\big)\, \underbrace{iFT\Big[\underbrace{\exp\big(h\widehat{D}_\omega\big)\, \tilde{A}(z,\omega)}_{\text{Dispersion operator in frequency domain}}\Big]}_{\text{Nonlinear operator in time domain}}\Big\}$$

Improved accuracy of the split-step Fourier

Can improve the accuracy by including the effect of nonlinearity in the middle of the segment (at $h/2$) rather than at the segment boundary – ***symmetrised SSFM***

$$A(z+h,T) \cong \exp\left[\frac{h}{2}\widehat{D}_T\right]\exp\left[h\widehat{N}_T\right]\exp\left[\frac{h}{2}\widehat{D}_T\right]A(z,T)$$

The method can be made to run faster by noting the identical $h/2$ products

- Except for the 1st and last dispersive steps, all other steps can be carried over the entire length h
- Reduces the required number of FFT

$$A(L,T) \cong \exp\left[-\frac{h}{2}\widehat{D}_T\right]\left[\prod_{m=1}^{M}\exp\left[h\widehat{D}_T\right]\exp\left[h\widehat{N}_T\right]\right]\exp\left[\frac{h}{2}\widehat{D}_T\right]A(0,T)$$

Symmetrised SSFM - summary

The fiber length is divided into a large number of segments (do not need to be spaced equally)

Optical pulse is propagated from segment to segment:

- 1) optical field $A(z,T)$ is first propagated over $h/2$ with dispersion only using FFT algorithm
- 2) at $z + h/2$, the field is multiplied by nonlinear term
  - Represents the effect of nonlinearity over entire h segment (lumped nonlinearity)
- Field is propagated the remaining $h/2$ with dispersion only: get $A(z+h,T)$

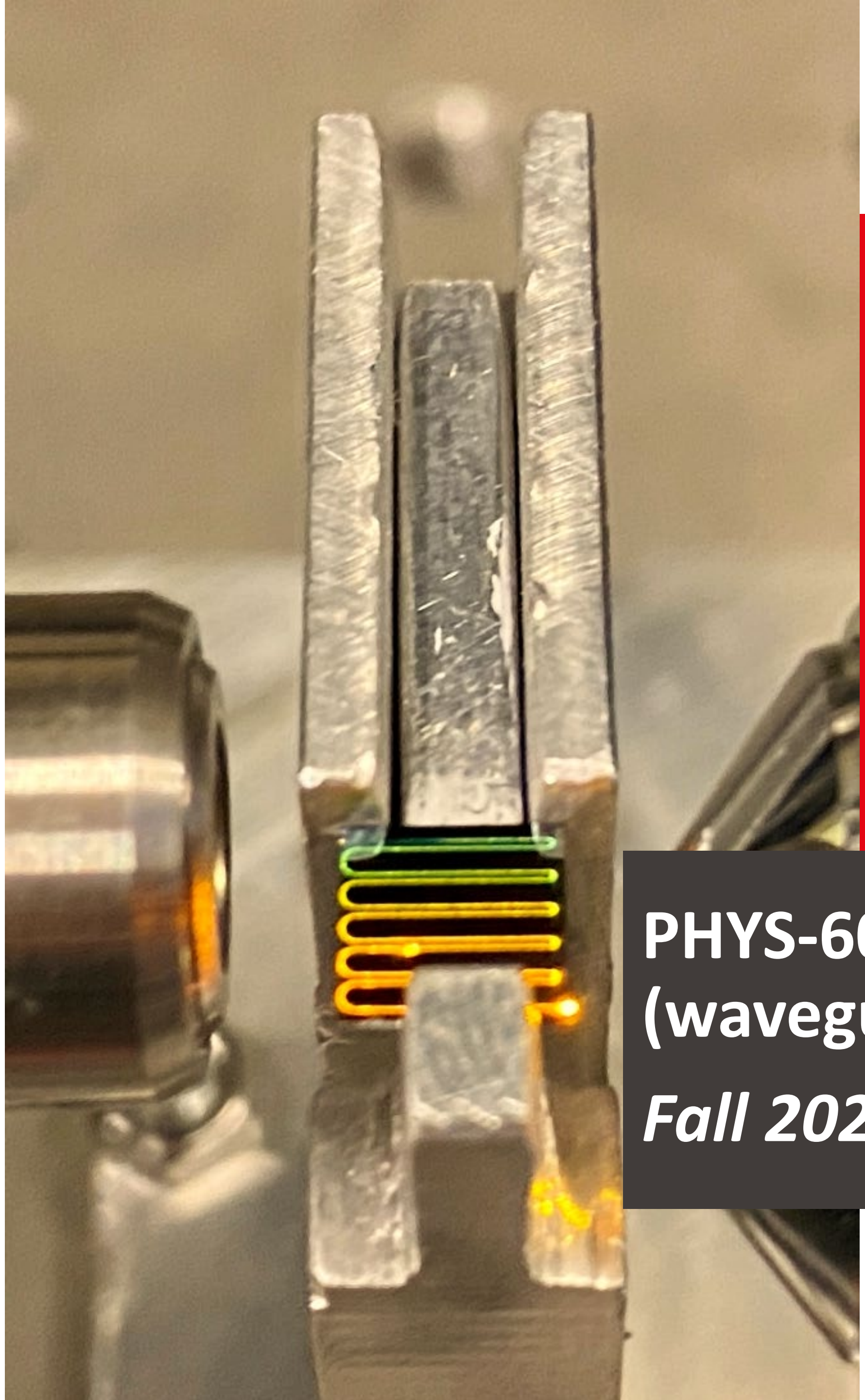
You can also perform the algorithm by interchanging the dispersion and nonlinear operator.

- Both provide the same accuracy and are easy to implement in practice
- The step sizes in z and T however need to be selected carefully to ensure accuracy

See Example of Numerical Code for the NLS Equation – Agrawal Nonlinear fiber Optics Appendix B
Code is also easily found for your to play around.

Examples will be shown in the Supercontinuum module !

EPFL



Optical Kerr effect

Module 7 – Multiwavelength effects (XPM and FWM)

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Nonlinear propagation

Reminder

Given the dominant 3rd order nonlinear polarization in most optical waveguiding systems:

$$\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)} \mathbf{E}^3$$

And single harmonic field:

$$\mathbf{E}(t) = \mathrm{Re}[E(t)\exp(-j\omega t)] = \frac{1}{2}[E(t)\exp(-j\omega t) + c.c]$$

We had:

$$\mathbf{P}_{NL}(t) = \frac{1}{2}\left[P_{NL,\omega}\exp(-j\omega t) + P_{NL,3\omega}\exp(-j3\omega t) + c.c\right]$$

$$P_{NL,\omega} = \frac{3}{4}\varepsilon_0 \chi^{(3)} E^2 E^*$$ **ω frequency source**

$$P_{NL,3\omega} = \frac{1}{4}\varepsilon_0 \chi^{(3)} E^3$$ **3ω frequency source**

Nonlinear propagation – *two optical waves at different wavelengths*

$$\mathbf{E}(t) = \frac{1}{2}[E_1(t)\exp(-j\omega_1 t) + E_2(t)\exp(-j\omega_2 t) + c.c]$$

The resulting nonlinear polarization is:

$$\mathbf{P}_{NL}(t) = \frac{1}{2}\left[P_{NL,\omega_1}e^{(-j\omega_1 t)} + P_{NL,\omega_2}e^{(-j\omega_2 t)} + P_{NL,2\omega_1-\omega_2}e^{(-j(2\omega_1-\omega_2)t)} + P_{NL,2\omega_2-\omega_1}e^{(-j(2\omega_2-\omega_1)t)} + c.c\right]$$

With:

$$P_{NL,\omega_1} = \frac{3}{4}\varepsilon_0\chi^{(3)}[|E_1|^2 + 2|E_2|^2]E_1$$

$$P_{NL,\omega_2} = \frac{3}{4}\varepsilon_0\chi^{(3)}[|E_2|^2 + 2|E_1|^2]E_2$$

$\Rightarrow$ *SPM and XPM (no phase matching required)*

$$P_{NL,2\omega_1-\omega_2} = \frac{3}{4}\varepsilon_0\chi^{(3)}E_1^2E_2^*$$

$$P_{NL,2\omega_2-\omega_1} = \frac{3}{4}\varepsilon_0\chi^{(3)}E_2^2E_1^*$$

$\Rightarrow$ *Degenerate FWM (phase matching required)*

Try this at home

Nonlinear propagation – *two optical pulses at different wavelengths*

Let's consider only the SPM and XPM terms:

$$P_{NL,\omega_1} = \frac{3}{4}\varepsilon_0\chi^{(3)}[|E_1|^2 + 2|E_2|^2]E_1 \quad \Longrightarrow \quad \Delta\chi = \frac{3}{4}\chi^{(3)}[|E_1|^2 + 2|E_2|^2]$$

The resulting refractive index change and associated phase shift is therefore:

$$\Delta n_{r,\omega_1} = \frac{\mathrm{Re}[\Delta\chi]}{2n} = \frac{3}{8n}\mathrm{Re}\left[\chi^{(3)}\right][|E_1|^2 + 2|E_2|^2] = n_2[I_1 + 2I_2]$$

$$\varphi_{NL_{max},\omega_1} = \gamma(P_1 + 2P_2)L_{eff}$$

Wave mixing in a 3rd order nonlinear medium

In a medium subject to 3rd order nonlinearity, such as Kerr effect, 3 waves with distinct frequencies cannot couple without the intercession of a 4th wave

Let's therefore consider the simultaneous propagation of 3 waves:

$$\mathbf{E}(t) = \frac{1}{2} \sum_{q=\pm1,\pm2,\pm3} E_q \exp\left(-j\omega_q t\right) \qquad \left(\text{with } \omega_{-q} = -\omega_q \text{ and } E_{-q} = E_q^*\right)$$

The nonlinear polarization can be expressed as the sum of 6^3 = 216 terms:

$$\mathbf{P}_{NL}(t) = \frac{1}{2} \sum_{q,r,l=\pm1,\pm2,\pm3} P_{NL,\omega_q+\omega_r+\omega_l} \exp\left(-j\left(\omega_q + \omega_r + \omega_l\right)t\right)$$

$$\mathbf{P}_{NL}(t) = \frac{1}{8} \varepsilon_0 \chi^{(3)} \sum_{q,r,l=\pm1,\pm2,\pm3} E_q E_r E_l \exp\left(-j\left(\omega_q + \omega_r + \omega_l\right)t\right)$$

Four-wave mixing examples

Example 1: A solution for the nonlinear polarization oscillating at $(\omega_1 + \omega_2 - \omega_3)$ arises from 6 possible permutations of $\{q, r, l\}$:

$$\frac{1}{2}\left[P_{NL,\omega_1+\omega_2-\omega_3} \exp(-j(\omega_1 + \omega_2 - \omega_3)t) + c.c.\right]$$

$$\Longrightarrow P_{NL,\omega_1+\omega_2-\omega_3} = \frac{6}{4}\varepsilon_0\chi^{(3)}E_1E_1E_3^*$$

Example 2: degenerate FWM occurs when two or more frequencies are equal (eg $\omega_2 = \omega_3$)

$$\frac{1}{2}\left[P_{NL,2\omega_2-\omega_1} \exp(-j(2\omega_2 - \omega_1)t) + c.c.\right]$$

$$\Longrightarrow P_{NL,2\omega_2-\omega_1} = \frac{M}{4}\varepsilon_0\chi^{(3)}E_2E_2E_1^*$$

M is the multiplicity factor.

Complete description of FWM

To get a better description of FWM we must include the propagation constants β to highlight the phase matching rules.

$$\mathbf{E}(t) = \frac{1}{2}\sum_{j=1}^{3} E_j \exp\left[j\left(-\omega_j t + \beta_{\omega_j} z\right)\right] + c.c.$$

We express the nonlinear polarization in the same form:

$$\mathbf{P}_{NL}(t) = \frac{1}{8}\varepsilon_0 \chi^{(3)} \sum_{q,r,l=\pm 1,\pm 2,\pm 3} E_q E_r E_l \exp\left(-j(\omega_q + \omega_r + \omega_l)t + j\left(\beta_{\omega_q} + \beta_{\omega_r} + \beta_{\omega_l}\right)\right)$$

Four-wave mixing examples

Example 1: A solution for the nonlinear polarization oscillating at $(\omega_1 + \omega_2 - \omega_3)$

$$\frac{1}{2}\left[P_{NL,\omega_1+\omega_2-\omega_3}\exp(-j(\omega_1+\omega_2-\omega_3)t) + c.c.\right]$$

$$\Longrightarrow P_{NL,\omega_1+\omega_2-\omega_3} = \frac{6}{4}\varepsilon_0\chi^{(3)}E_1E_1E_3^*$$

Energy conservation:

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

Momentum conservation (phase matching condition):

$$\beta_{\omega_1} + \beta_{\omega_2} = \beta_{\omega_3} + \beta_{\omega_4}$$

Phase and frequency matching

$$\omega_1 + \omega_2 + \omega_3 = \omega_4$$
$$\Delta k = \beta_{\omega_1} + \beta_{\omega_2} + \beta_{\omega_3} - \beta_{\omega_4} = 0$$

- Three photons transfer their energy to a single photon at frequency ω_4.
- When $\omega_1 = \omega_2 = \omega_3$, this is third harmonic generation
- Difficult to satisfy the phase matching condition for such process in optical fiber

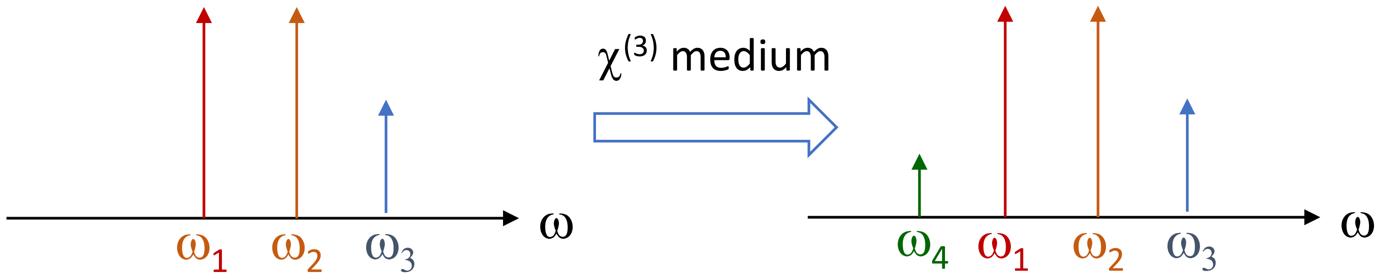
$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$
$$\Delta k = \beta_{\omega_4} + \beta_{\omega_3} - \beta_{\omega_1} - \beta_{\omega_2} = 0$$

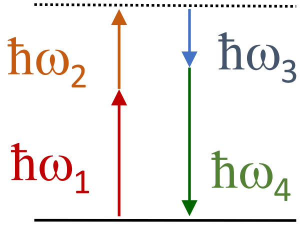
- Two photons at ω_1 and ω_2 are annihilated, while two photons at ω_3 and ω_4 are created simultaneously.
- When $\omega_1 = \omega_2 = \omega_3$, this is self phase modulation
- When $\omega_2 = \omega_3$, this is cross phase modulation
- When $\omega_1 = \omega_2$, this process is degenerate FWM

Typical four wave mixing processes

General case: $\omega_1 \neq \omega_2$

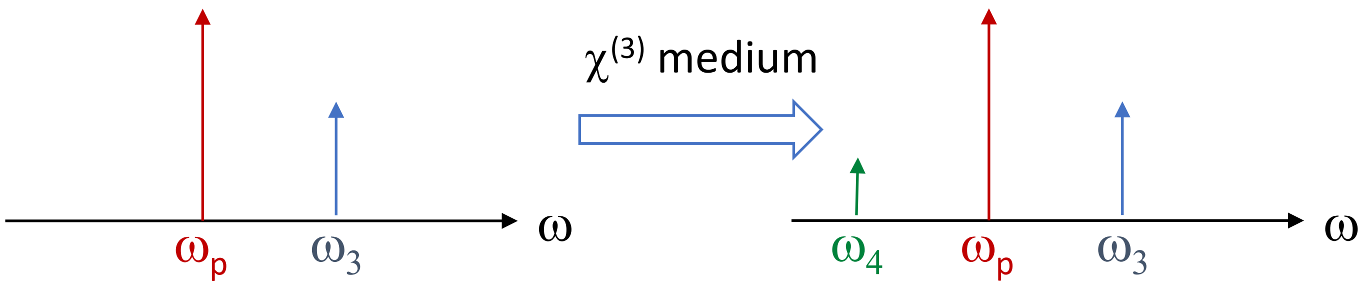
- Launch 2 beams for FWM to occur
- If have a signal at ω_3, a new frequency will be created at ω_4

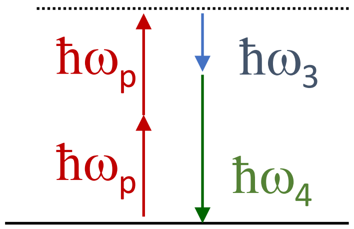




In the degenerate case: $\omega_1 = \omega_2 = \omega_p$

- FWM is initiated from a single beam



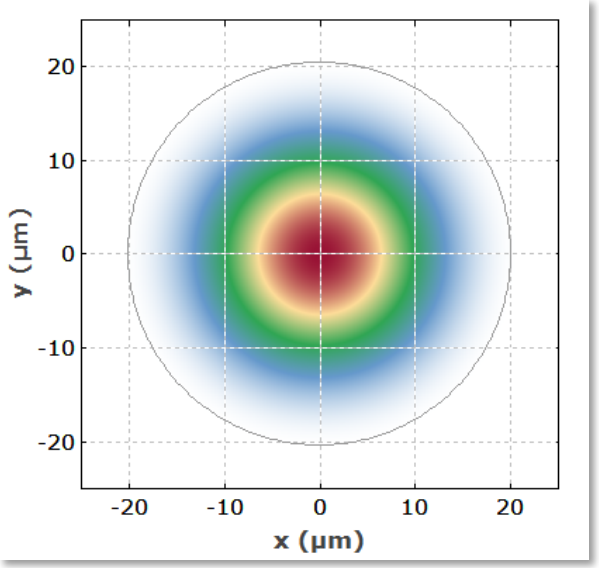


Coupled NLS equations

Reminder

Start from the wave equation: $$\nabla^2\mathbf{E} - \frac{1}{c}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}_L}{\partial t^2} + \mu_0\frac{\partial^2\mathbf{P}_{NL}}{\partial t^2}$$

- Substitute $\mathbf{E}$, $\mathbf{P}_\mathrm{L}$ and $\mathbf{P}_\mathrm{NL}$ and neglect time dependence of the field components E_j (j = 1 – 4)
- Include spatial dependence $E_\mathrm{j}(\mathbf{r}) = F_\mathrm{j}(\mathrm{x, y})A_\mathrm{j}(\mathrm{z})$
- Integrate over spatial mode profile and get the evolution of the amplitude $A_\mathrm{j}(\mathrm{z})$ inside the waveguide



$$f_{jk} = \frac{\iint_{-\infty}^{\infty}\left|F_j(x,y)\right|^2|F_k(x,y)|^2dxdy}{\left(\iint_{-\infty}^{\infty}\left|F_j(x,y)\right|^2dxdy\iint_{-\infty}^{\infty}|F_k(x,y)|^2dxdy\right)}$$

$$f_{ijkl} = \frac{\left\langle F_i^*F_j^*F_kF_l\right\rangle}{\left[\langle|F_i|^2\rangle\left\langle\left|F_j\right|^2\right\rangle\langle|F_k|^2\rangle\langle|F_l|^2\rangle\right]^{\frac{1}{2}}}$$

with $\langle S\rangle = \iint\limits_{-\infty}^{\infty} S(x,y)dxdy$

Coupled NLS equations

$$\frac{dA_1}{dz} = j\frac{n_2\omega_1}{c}\left[\left(f_{11}|A_1|^2 + 2\sum_{k\neq 1} f_{1k}|A_k|^2\right)A_1 + 2f_{1234}A_2^*A_3A_4e^{j\Delta kz}\right]$$

$$\frac{dA_2}{dz} = j\frac{n_2\omega_2}{c}\left[\left(f_{22}|A_2|^2 + 2\sum_{k\neq 2} f_{2k}|A_k|^2\right)A_2 + 2f_{2134}A_1^*A_3A_4e^{j\Delta kz}\right]$$

$$\frac{dA_3}{dz} = j\frac{n_2\omega_3}{c}\left[\left(f_{33}|A_3|^2 + 2\sum_{k\neq 3} f_{3k}|A_k|^2\right)A_3 + 2f_{3412}A_4^*A_1A_2e^{j\Delta kz}\right]$$

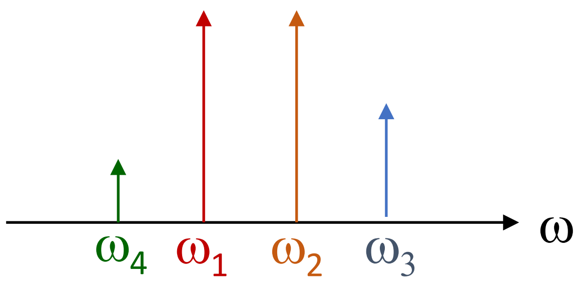
$$\frac{dA_4}{dz} = j\frac{n_2\omega_4}{c}\left[\left(f_{44}|A_4|^2 + 2\sum_{k\neq 4} f_{4k}|A_k|^2\right)A_4 + 2f_{4312}A_3^*A_1A_2e^{j\Delta kz}\right]$$

$$\Delta k = \beta_{\omega_4} + \beta_{\omega_3} - \beta_{\omega_1} - \beta_{\omega_2}$$

Coupled wave equations - simplifications

We assume:

- Intense and un-depleted pump waves A_1 and A_2
- Identical transverse modes: overlap integrals $f_{ijkl} \approx f_{jk} \approx 1/A_{eff}$
- Identical nonlinear coefficient γ.



The equations for the pump fields are easily solved based on these assumption:

$$\frac{dA_1}{dz} = j\gamma(|A_1|^2 + 2|A_2|^2)A_1 \qquad A_1(z) = \sqrt{P_1}\exp[j\gamma(P_1^2 + 2P_2^2)z]$$

$$\frac{dA_2}{dz} = j\gamma(|A_2|^2 + 2|A_1|^2)A_2 \qquad A_2(z) = \sqrt{P_2}\exp[j\gamma(P_2^2 + 2P_1^2)z]$$

In un-depleted approximation, the pump waves only acquire a phase shift as a result of SPM and XPM

Coupled wave equations - simplifications

Using the obtained expression for A_1 and A_2 in the remaining 2 equations we get:

$$\frac{dA_3}{dz} = 2j\gamma\left[(P_1 + P_2)A_3 + \sqrt{P_1P_2}A_4^* e^{-j\theta}\right]$$

$$\frac{dA_4^*}{dz} = -2j\gamma\left[(P_1 + P_2)A_4^* + \sqrt{P_1P_2}A_3 e^{j\theta}\right]$$

- With: $\theta = [\Delta k - 3\gamma(P_1 + P_2)]z$ and $\Delta k = \beta_{\omega_4} + \beta_{\omega_3} - \beta_{\omega_1} - \beta_{\omega_2}$

Show this at home

To solve, we substitute: $B_j = A_j \exp[-2j\gamma(P_1 + P_2)z]$

$$\frac{dB_3}{dz} = 2j\gamma\sqrt{P_1P_2}\exp(-j\kappa z)\, B_4^*$$

$$\frac{dB_4^*}{dz} = -2j\gamma\sqrt{P_1P_2}\exp(j\kappa z)\, B_3$$

$$\kappa = \Delta k + \gamma(P_1 + P_2)$$

Coupled wave equations - simplifications

By combining the 2 resulting equations we get:

$$\frac{d^2B_3}{dz^2} + j\kappa\frac{dB_3}{dz} - (4\gamma^2P_1P_2)B_3 = 0$$

The same equation is obtained for $B_4{}^*$ and the general solution takes the form:

$$B_3(z) = (a_3e^{gz} + b_3e^{-gz})\exp\left(-j\kappa\frac{z}{2}\right)$$

$$B_4(z) = (a_4e^{gz} + b_4e^{-gz})\exp\left(j\kappa\frac{z}{2}\right)$$

- a_3, b_3, a_4 and b_3 are determined from boundary conditions

The most important parameter is g, called parametric gain: $g = \sqrt{(\gamma P_0 r)^2 - \left(\frac{\kappa}{2}\right)^2}$

- with $P_0 = \frac{P_1 + P_2}{2}$ $\quad r = \frac{\sqrt{P_1P_2}}{P_0}$

Illustration with degenerate case

We consider the degenerate case where the pump fields cannot be distinguished – single pump

$$P_1 = P_2 = P_0 \quad \text{and} \quad r = 1$$

$$\kappa = \Delta k + 2\gamma P_0$$

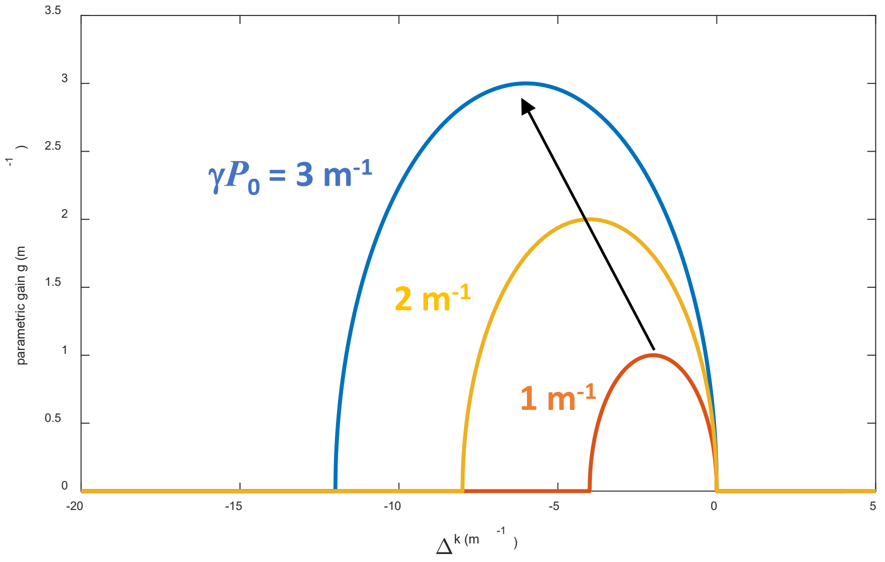
$$g = \sqrt{(\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2}$$

Gain exists over a range given by: $\quad (\gamma P_0)^2 - \left(\frac{\kappa}{2}\right)^2 > 0$

$$-4\gamma P_0 < \Delta k < 0$$

Maximum parametric gain is: $\quad g = (\gamma P_0)$

- Occurs for $\kappa = 0 \; or \; at \; \Delta k = -2\gamma P_0$



Phase matching consideration

As just seen, the maximum gain for the underlying FWM process is obtained when κ = 0

- Note that the gain spectrum indicates that significant FWM can still occur even if the phase matching is not perfect

How to satisfy phase matching $\kappa \approx 0 \implies \Delta k_M + \Delta k_W + \Delta k_{NL} \approx 0$

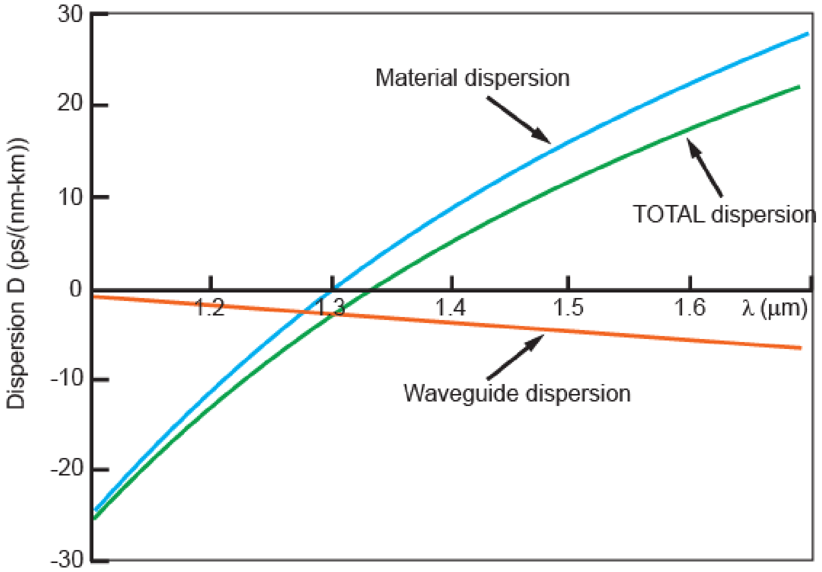
Consider effective refractive index at ω_j $\tilde{n}_j = n_j + \Delta n_j$

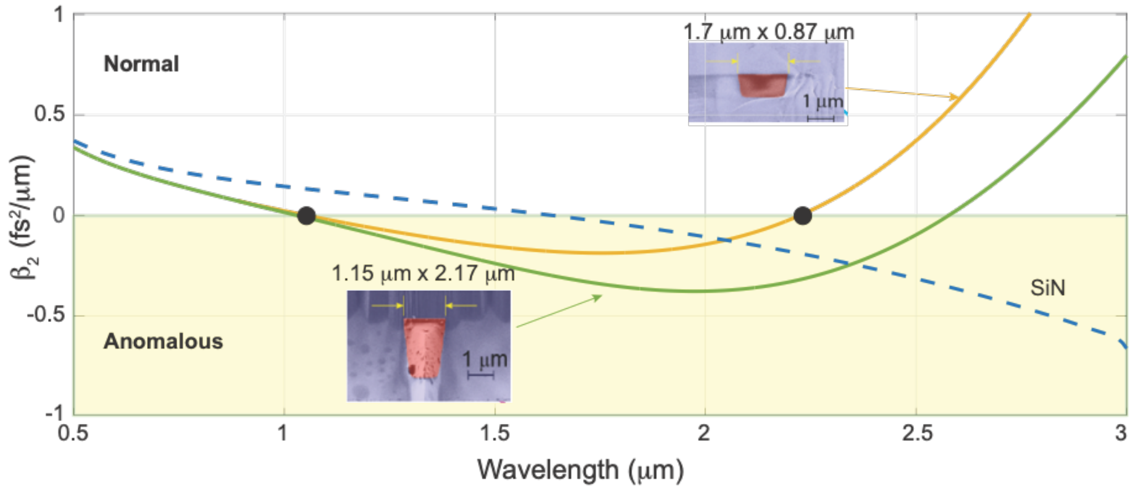
Mismatch due to material dispersion: $\Delta k_M = n_4 \frac{\omega_4}{c} + n_3 \frac{\omega_3}{c} - 2n_1 \frac{\omega_1}{c}$

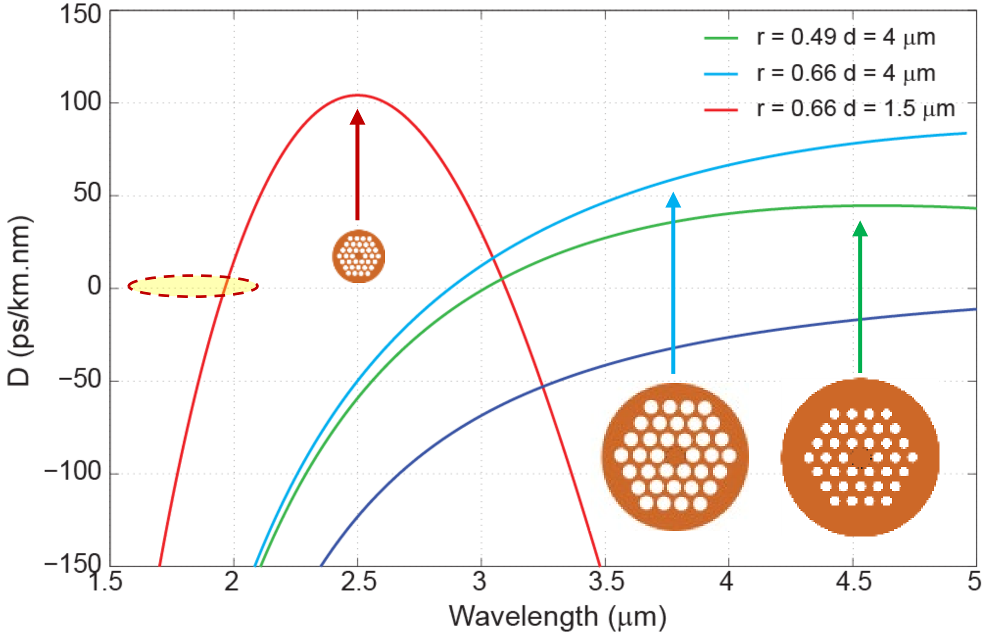
Mismatch due to material dispersion: $\Delta k_W = \Delta n_4 \frac{\omega_4}{c} + \Delta n_3 \frac{\omega_3}{c} - 2\Delta n_1 \frac{\omega_1}{c}$

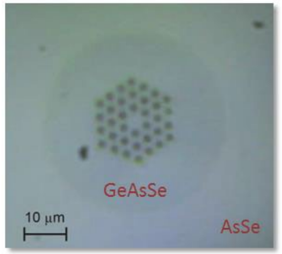
Mismatch due to nonlinear effects: $\Delta k_{NL} = 2\gamma P_0$

Effective dispersion (material + waveguide)









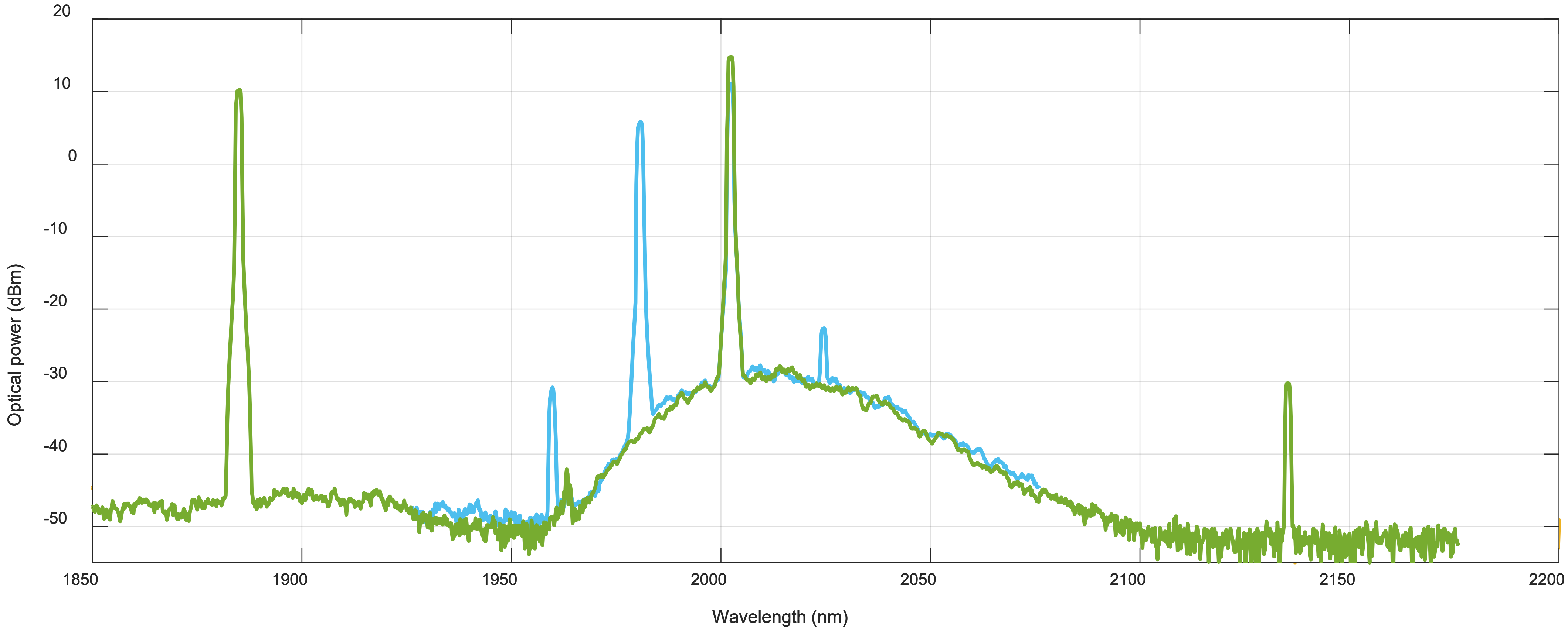
Phase matching consideration

$\Longrightarrow \Delta k_M + \Delta k_W + \Delta k_{NL} \approx 0$ at least one of the contribution should be negative

Let's consider the overall dispersion due to the effective refractive index $\Delta k = \beta_{\omega_4} + \beta_{\omega_3} - 2\beta_{\omega_1}$

- We can express it in terms of frequency shift $\Omega_s = |\omega_1 - \omega_3| = |\omega_1 - \omega_4|$
- Use the expansion $\beta_\omega = \beta_0 + \beta_1(\omega - \omega_1) + \frac{1}{2}\beta_2(\omega - \omega_1)^2 + \frac{1}{6}\beta_3(\omega - \omega_1)^3 + \cdots$
- Get $\Delta k = \beta_2 \Omega_s^2 + \frac{\beta_4}{12}\Omega_s^4$ ***Show this at home***

Experimental illustration of FWM



Quiz

We consider degenerate FWM and want to use a pump positioned at 1500 nm. We want to mix it with a signal wavelength at 1510 nm

We have a fiber with a nonlinear coefficient γ = 5 $W^{-1}km^{-1}$, β_2 = -20 ps^2/km and negligible β_4. What should be the pump power P_0 such that phase matching be satisfied?

a) 1 .3 W

b) 138.5 W

c) 256 mW

d) 69 W